

# Double logarithms, $\ln^2(1/x)$ , and the NLO DGLAP evolution for the non-singlet component of the nucleon spin structure function, $g_1$ .

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**Abstract:** Theoretical predictions show that at low values of Bjorken  $x$  the spin structure function,  $g_1$  is influenced by large logarithmic corrections,  $\ln^2(1/x)$ , which may be predominant in this region. These corrections are also partially contained in the NLO part of the standard DGLAP evolution. Here we calculate the non-singlet component of the nucleon structure function,  $g_1^{NS} = g_1^p - g_1^n$ , and its first moment, using a unified evolution equation. This equation incorporates the terms describing the NLO DGLAP evolution and the terms contributing to the  $\ln^2(1/x)$  resummation. In order to avoid double counting in the overlapping regions of the phase-space, a unique way of including the NLO terms into the unified evolution equation is proposed. The scheme-independent results obtained from this unified evolution are compared to the NLO fit to experimental data, GRSV'2000. Analysis of the first moments of  $g_1^{NS}$  shows that the unified evolution including the  $\ln^2(1/x)$  resummation goes beyond the NLO DGLAP analysis. Corrections generated by double logarithms at low  $x$  influence the  $Q^2$ -dependence of the first moments strongly.

## 1. Introduction

During the last years the interest in physics of polarized nucleon has increased significantly. Experimental data describing the structure of the nucleon are now available in various ranges of the momentum transfer,  $Q^2$ , and the Bjorken variable,  $x$ . New experiments emerge, and new data are expected to be available in the next future. The data from the region of low  $x$  will be of special importance, since they may improve the estimation of the quark contribution to the total nucleon spin.

The previous data obtained by the EMC collaboration [1] showed that the total participation of quarks in the proton spin was very small. This contradicted theo-

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retical predictions, obtained from the Ellis-Jaffe sum rule. That sum rule expressed the moments of quark distributions in terms of the nucleon axial coupling constants. Following the Ellis-Jaffe sum rule, the quarks should participate in about one-fifth of the total spin of nucleon. The discrepancy between the theoretical expectations and the experimental data has been often referred as the "proton spin crisis".

There have been several explanations proposed to clarify this. One of them refers to the not-yet-measured domain of very low values of Bjorken  $x$ ,  $x < 10^{-3}$ . Theoretical predictions show that at low  $x$  the structure function of the polarized nucleon,  $g_1(x, Q^2)$ , is influenced by large logarithmic corrections,  $\ln^2(1/x)$ , [2, 3]. As a consequence, large contributions to the moments of the structure functions from this region are expected.

Analysis of the nucleon structure function at low  $x$ , including the resummation of the logarithmic corrections, was performed in [4, 2, 3, 5–8]. The logarithmic corrections were introduced for the unintegrated parton distributions in Refs. [4, 9–11]. Those corrections originated from the ladder and the non-ladder bremsstrahlung diagrams [2, 3, 12, 13].

The recursive equations for resummation of the logarithmic corrections were formulated. Afterwards, these evolution equations were completed with the DGLAP evolution terms calculated at the LO accuracy [9, 4]. Including the DGLAP evolution was necessary for accurate description of the structure functions in the region of moderate and large values of  $x$ .

Since the phase-space domains for the logarithmic and the DGLAP contributions overlapped partially, subtractions from the evolution kernels were made in order to avoid double counting in those regions. It was shown that including the  $\ln^2(1/x)$  terms into the evolution equations generated more singular behaviour of  $g_1(x, Q^2)$  at low  $x$ , than that obtained after the standard LO DGLAP evolution. This led to the conclusion that the pure LO DGLAP evolution might be not complete at low  $x$ .

On the other hand, the description of polarized data based exclusively on the DGLAP evolution has improved after including the DGLAP evolution terms calculated at the NLO accuracy [14, 15]. These predictions described data accurately even in the region of low  $x$  ( $x > 10^{-4}$ ). However, the relation between the NLO corrections and the  $\ln^2(1/x)$  resummation has not been clarified yet. It was expected that at low  $x$  the NLO corrections contained in part the logarithmic terms but it was not clear to what extent those corrections overlapped.

Here we analyze the interplay between the NLO corrections and the corrections obtained from the resummation of double logarithms,  $\ln^2(1/x)$ . We formulate unified

evolution equations for the *unintegrated* parton distributions, where we include the NLO DGLAP terms. Parton distributions are not physical observables and, in general, they depend on the factorization scheme applied to the NLO analysis (cf. [5, 6, 16]). They yield (scheme-independent) physical observables after a convolution with the Wilson coefficient functions (cf. Eq. (5)).

In this study we will consider the non-singlet component of the nucleon structure function,  $g_1^{NS} = g_1^p - g_1^n$ . This component possesses a property that it does not depend on the factorization scheme applied within the family of  $\overline{MS}$ -like factorization schemes [15, 17]. This occurs at the level of parton distributions already.

We formulate the equations including the logarithmic corrections,  $\ln^2(1/x)$ , and the NLO corrections at the  $\overline{MS}$  factorization scheme. We introduce a unique way of performing subtractions in the evolution kernels, so as to avoid double counting between the double logarithmic terms and the NLO ones in the overlapping regions of the phase-space. Predictions for the non-singlet component of the nucleon spin structure function,  $g_1^{NS}(x, Q^2)$  and its first moment are then obtained. These results are compared with the NLO fit to the experimental data, GRSV'2000, [18]. Finally, our conclusions are listed.

## 2. Unintegrated parton distributions and DGLAP evolution

Differential equations describing the evolution of standard (integrated) parton distributions may be transformed into integral equations, if introducing the unintegrated parton distributions. These distributions,  $f_i(x, Q^2)$ , are defined as:

$$\Delta q_i(x, Q^2) = \Delta q_i^{(0)}(x) + \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} f_i(x, k^2), \quad (1)$$

where  $\Delta q_i(x, Q^2)$  denote the integrated parton distributions with  $\Delta q_i^{(0)}(x)$  describing the contributions coming from the non-perturbative region,  $Q^2 < k_0^2$ . The cutoff  $k_0^2$  is usually  $\sim 1 \text{ GeV}^2$ .

The standard DGLAP equations written for the evolution of the integrated non-singlet parton distribution,  $\Delta q_{NS}$ ,

$$\frac{d}{d \ln Q^2} \Delta q_{NS}(x, Q^2) = \widetilde{\alpha}_s(Q^2) (\Delta P \otimes \Delta q_{NS})(x, Q^2), \quad (2)$$

with  $\widetilde{\alpha}_s(Q^2) \equiv \frac{\alpha_s(Q^2)}{2\pi}$ , transform to the following integral equations:

$$f_{NS}(x, Q^2) = \widetilde{\alpha}_s(Q^2) (\Delta P \otimes \Delta q_{NS}^{(0)})(x) + \widetilde{\alpha}_s(Q^2) \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} (\Delta P \otimes f_{NS})(x, k^2), \quad (3)$$

if the relation (1) is applied. In particular, we have:  $\frac{d}{d \ln Q^2} \Delta q_i(x, Q^2) = f_i(x, Q^2)$  at the LHS of Eq. (2), and, substituting (1) to the RHS of Eq. (2), one obtains the RHS of Eqs. (3).

Coefficients,  $\Delta P$ , are the splitting functions calculated for the polarized deep inelastic scattering. Symbol  $\otimes$  denotes the integral convolution of two functions,  $(f \otimes g)(x) = \int_0^1 dy \int_0^1 dz \delta(x - yz) f(y) g(z)$ .

A complete description of the DGLAP evolution of polarized parton distributions at the NLO accuracy was performed primarily in one of the  $\overline{MS}$  factorization scheme. The splitting functions describing the evolution of the non-singlet components and the singlet ones were firstly derived in [19,20], and they are listed in detail in the Appendix of Ref. [14]. The equations for the DGLAP evolution at the NLO order are structurally identical with Eq. (2). The only difference is that the non-singlet distribution,  $\Delta q_{NS}$ , separates into two parts: the valence one,  $\Delta q_{NS}^+ = \Delta q - \Delta \bar{q}$ , and the asymmetric one,  $\Delta q_{NS}^- = \Delta q + \Delta \bar{q} - (\Delta q' + \Delta \bar{q}')$ , which have to be evolved with different kernels. Here we will evolve the non-singlet isospin-3 parton component,

$$\Delta q_{NS,3} = \Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d}, \quad (4)$$

using the asymmetric  $\overline{MS}$  evolution kernels [14]. This component contributes to  $g_1^{NS}$ .

NLO DGLAP evolution of parton distributions depends on the factorization scheme applied. In general, scheme dependence disappears in physical observables, e.g. in the structure functions,  $g_1(x, Q^2)$ , which are combinations of the parton distributions convoluted with the Wilson coefficient functions,  $\Delta C_{q,g}(x)$ :

$$\begin{aligned} g_1(x, Q^2) = & \frac{1}{2} \sum_{q=1}^{N_f} e_q^2 \left\{ \left( (\delta + \widetilde{\alpha}_s(Q^2) \Delta C_q) \otimes (\Delta q + \Delta \bar{q}) \right) (x, Q^2) \right. \\ & \left. + \widetilde{\alpha}_s(Q^2) \frac{1}{N_f} (\Delta C_g \otimes \Delta g)(x, Q^2) \right\}, \end{aligned} \quad (5)$$

where  $\delta$  is the Dirac  $\delta$ -function,  $\delta(x - 1)$ . The coefficient functions calculated at the  $\overline{MS}$  scheme [19] are taken from Ref. [14]. The constant  $N_f$  denotes the number of active flavours. Here,  $N_f = 3$ .

In case of the non-singlet parton distributions, the dependence on the factorization scheme disappears already at the level of the integrated parton distributions,  $\Delta q_{NS}^\pm$ , within the set of the  $\overline{MS}$ -like schemes [15,17]. The non-singlet component of the nucleon structure function is then, obviously, also scheme-independent, and it reads:

$$g_1^{NS}(x, Q^2) = \frac{1}{6} \left( (\delta + \widetilde{\alpha}_s(Q^2) \Delta C_q) \otimes \Delta q_{NS,3} \right) (x, Q^2) \quad (6)$$

### 3. Unified evolution equations including $\ln^2(1/x)$ resummation.

A detailed study of the nucleon structure function at low  $x$  was performed in [4]. There a complete resummation of the double logarithmic corrections,  $\ln^2(1/x)$ , was performed for the unintegrated parton distributions. Those corrections originated from summing up the ladder and the non-ladder bremsstrahlung diagrams [2, 3] with the requirement of ordering the ratios,  $k_n^2/x_n$ , for exchanged partons. The ratio,  $k_n^2/x_n$ , corresponded to the Sudakov variable,  $\beta$ , the momentum,  $k_n$ , was the transverse momentum, and the fraction,  $x_n$ , was the longitudinal momentum fraction of the  $n$ th parton exchanged. The recursive equations for the resummation of the logarithmic corrections were then formulated. The integrated parton distributions were obtained from the unintegrated ones, using a relation similar to (1):

$$\Delta q_i(x, Q^2) = \Delta q_i^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f_i(x' = x(1 + \frac{k^2}{Q^2}), k^2), \quad (7)$$

where the phase-space was extended to  $W^2 = Q^2(1/x - 1)$  corresponding to the total energy squared measured in the center-of-mass frame.

Following [4], we combine the NLO DGLAP evolution and the logarithmic resummation into a unified evolution equation, including both the DGLAP kernels and the  $\ln^2(1/x)$  evolution kernels:

$$\begin{aligned} f_{NS}(x, Q^2) &= \widetilde{\alpha}_s(Q^2)(\Delta P \otimes \Delta q_{NS}^{(0)})(x) + \widetilde{\alpha}_s(Q^2) \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} (\Delta P_{reg} \otimes f_{NS})(x, k^2) \\ &\quad \text{( DGLAP )} \\ &+ \widetilde{\alpha}_s(Q^2) \frac{4}{3} \int_{x'}^1 \frac{dz}{z} \int_{Q^2}^{Q^2/z} \frac{dk^2}{k^2} f_{NS}\left(\frac{x'}{z}, k^2\right) \\ &\quad \text{( Ladder )} \\ &- \widetilde{\alpha}_s(Q^2) \int_{x'}^1 \frac{dz}{z} \left( \left[ \frac{\tilde{\mathbf{F}}_8}{\omega^2} \right](z) \frac{\mathbf{G}_0}{2\pi^2} \right)_{qq} \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} f_{NS}\left(\frac{x'}{z}, k^2\right) \\ &- \widetilde{\alpha}_s(Q^2) \int_{x'}^1 \frac{dz}{z} \int_{Q^2}^{Q^2/z} \frac{dk^2}{k^2} \left( \left[ \frac{\tilde{\mathbf{F}}_8}{\omega^2} \right] \left( \frac{k^2}{Q^2} z \right) \frac{\mathbf{G}_0}{2\pi^2} \right)_{qq} f_{NS}\left(\frac{x'}{z}, k^2\right). \\ &\quad \text{(Non - ladder)} \end{aligned} \quad (8)$$

where  $x' = x(1 + \frac{k^2}{Q^2})$ . For a detailed form of the kernels see Appendix A. Matrices,  $\mathbf{F}_8$  and  $\mathbf{G}_0$ , represent octet partial waves and colour factors respectively. They are described in detail in Appendix A. Symbol  $\left[ \frac{\tilde{\mathbf{F}}_8}{\omega^2} \right](z)$  denotes the inverse Mellin transform of  $\mathbf{F}_8/\omega^2$ :

$$\left[ \frac{\tilde{\mathbf{F}}_8}{\omega^2} \right](z) = \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} z^{-\omega} \mathbf{F}_8(\omega)/\omega^2. \quad (9)$$

with the integration contour located to the right of the singularities of the function  $\mathbf{F}_8(\omega)/\omega^2$ .

Each of the kernels adds to the homogeneous part of the recursive evolution equation. In order to avoid double counting the NLO terms and the  $\ln^2(1/x)$  terms in the overlapping regions of the phase-space, we propose the following procedure of including the NLO terms into the evolution equations (cf. [4]). We divide the phase-space of these equations into two regions: (i)  $k_0^2 < k^2 < Q^2$  and (ii)  $Q^2 < k^2 < Q^2/z$ . In the region (i) we keep all terms generated by the (non-ladder) double logarithmic corrections and add only the regular part of (the LO) and NLO DGLAP terms,  $\Delta P_{reg}$ , i.e. this part which is not singular at  $z \rightarrow 0$  hence does not generate any  $\ln^2(1/x)$  contributions. In the region (ii) both LO and NLO DGLAP terms do not appear, and we have there only contributions from the ladder and the non-ladder double logarithmic terms. This procedure is unique, and it uses a proven result of Refs. [12, 13] that the  $\ln^2(1/x)$  resummation is complete after including the ladder and the non-ladder contributions.

While integrating the parton distributions over the extended phase-space,  $k^2 < W^2 = Q^2(1/x - 1)$  in (7), we may generate singular contributions similar to those appearing in the Wilson coefficient,  $\Delta C_q$ . Therefore, when calculating the non-singlet structure function,  $g_1^{NS}$ , we use only the regular part of the Wilson coefficient,

$$\Delta C_{q,reg} = \frac{4}{3} \left\{ (1+z^2) \left( \frac{\ln(1-z)}{(1-z)} \right)_+ - \frac{3}{2} \frac{1}{(1-z)_+} + 2 + z - \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-z) \right\}. \quad (10)$$

A full form of the Wilson coefficient was taken from Ref. [14]. Symbol  $()_+$  is defined by the following convolution. A function  $f(z)$  convoluted with a function  $(g(z))_+$  gives:

$$\int_0^1 dz f(z) (g(z))_+ = \int_0^1 dz (f(z) - f(1))g(z) \quad (11)$$

## 4. Results

We solved numerically the evolution equation (8) for the non-singlet isospin-3 parton component,  $f_{NS,3}$ .

Two different parameterizations of the non-perturbative parton distribution,  $\Delta q_{NS,3}^{(0)}(x)$ , at  $Q_0^2 = 1 \text{ GeV}^2$  were used: (i) the GRSV'2000 fit [18], and (ii) a simple "flat" input

$$\Delta p_i^{(0)}(x) = N_i(1-x)^{\eta_i} \quad (12)$$

with  $\eta_{u_v} = \eta_{d_v} = 3$ ,  $\eta_{\bar{u}} = \eta_{\bar{s}} = 7$  and  $\eta_g = 5$ . Normalization constants  $N_i$  were determined by imposing the LO Bjorken sum rule for  $\Delta u_v^{(0)} - \Delta d_v^{(0)}$ , and requiring

that the first moments of all other distributions are the same as those determined from the QCD analysis [21]. It was checked that parametrization (12) combined with the unified equations gave reasonable description of the SMC data on  $g_1^{NS}(x, Q^2)$  [9] and on  $g_1^p(x, Q^2)$  [22]. This fit was also used in [4, 23] to investigate the magnitude of the double logarithmic corrections to the spin structure function of proton and to its first moment.

The integrated parton distribution was then obtained by numerical integration of  $f_{NS,3}$ :

$$\Delta q_{NS,3}(x, Q^2) = \Delta q_{NS,3}^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f_{NS,3}(x' = x(1 + \frac{k^2}{Q^2}), k^2), \quad (13)$$

following Eq. (7).

Afterwards, we made a numerical convolution of  $\Delta q_{NS,3}$  with the Wilson coefficient function, in order to obtain the non-singlet nucleon structure function,  $g_1^{NS}$ ,

$$g_1^{NS}(x, Q^2) = \frac{1}{6} \left( (\delta + \widetilde{\alpha}_s(Q^2) \Delta C_{q,reg}) \otimes \Delta q_{NS,3} \right) (x, Q^2). \quad (14)$$

Figs. 1 a,b show the results.

After evolving the GRSV input, the differences between curves resulting from different approximations (DL+NLO, DL +LO, NLO, LO) were of the order of several percent at  $x > 10^{-3}$ . The curve representing the unified DL+NLO evolution underestimated the pure NLO results. The same occurred also for the evolution of the flat input (12), however, here the difference between the pure LO and other NLO results was described by the factor of 2 at  $x = 10^{-4}$ . In both cases the differences between the DL+NLO, DL+LO and NLO approximations manifested at very low  $x$ ,  $x \leq 10^{-4}$ . This was clearly the region, where  $\ln^2(1/x)$  resummation dominated. The results show that the shape of the input distribution influenced the evolution of the structure function strongly. We checked that the DL+LO and LO results obtained for  $g_1^{NS}$  are in a good agreement with the results of Ref. [4] (see Fig. 5 there).

Finally, we calculated the first moments of  $g_1^{NS}$ ,  $\Gamma_1^{NS}$ , starting from the inputs (i) and (ii). We estimated those moments in the following way. We integrated the non-singlet structure function,  $g_1^{NS}$ , over  $x$  in the region,  $10^{-4} < x < 1$ . Since at  $x < 10^{-4}$  the function  $g_1^{NS}$  was not known, we extrapolated  $g_1^{NS}$  into this region with the fit,  $x^{-\lambda}$ , and then integrated it over  $x$  at  $0 < x < 10^{-4}$ . It is worth mentioning that the exponent,  $\lambda$ , was found to be  $0.33 - 0.44$  at  $Q^2 = 2 - 10 \text{ GeV}^2$  with different inputs. That was in agreement with the recent QCD-based estimate from Ref. [24],  $\lambda = 0.40 \pm 0.29$ .

The asymptotic corrections, added to the previous results, changed them at about one percent at most. The moments obtained are plotted in Fig. 2. Results obtained from the GRSV and the flat inputs, after evolving them with the DL+NLO evolution equation, are similar. They follow but underestimate the theoretical predictions for  $\Gamma_1^{NS}$  obtained from the QCD analysis at the NNNLO accuracy (Bjorken's sum rule [14]). This discrepancy is less than 8 %. In contrast,  $\Gamma_1^{NS}$  estimated from numerical integration of the GRSV'2000 fit [18] follows Bjorken's sum rule calculated at the NLO accuracy,  $O(\alpha_s)$ . This clearly suggests that the unified evolution, DL+NLO, goes beyond the standard NLO DGLAP analysis, and that the corrections generated by double logarithms in the region of low  $x$  influence the behaviour of the first moments significantly.

## 5. Conclusions

We calculated the non-singlet component of the nucleon structure function,  $g_1^{NS} = g_1^p - g_1^n$ , and its first moment, using a unified evolution equation. This equation incorporated both the terms describing the NLO DGLAP evolution and the terms which contributed to the  $\ln^2(1/x)$  resummation. A unique way of including the NLO terms into the unified evolution equation was proposed, in order to avoid double counting in the overlapping regions of the phase-space. The results obtained from different approximations were similar, and the minor differences manifested at very low  $x$ ,  $x \leq 10^{-4}$ , where the  $\ln^2(1/x)$  resummation dominated. This effect depended on the shape of the non-perturbative input distribution,  $\Delta q_{NS}^{(0)}(x)$ . First moments obtained from different non-perturbative inputs evolved with DL+NLO evolution equations were similar. These moments followed Bjorken's sum rule calculated at the NNNLO accuracy. The discrepancy was at about 8 % at most. This shows that the unified evolution goes beyond the NLO DGLAP analysis. The  $\ln^2(1/x)$  corrections influence the  $Q^2$ -dependence of the first moments strongly.

Our results are scheme-independent within the  $\overline{MS}$ -like family of factorization schemes [15, 17].

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## Appendix A

Here a brief description of the evolution kernels of Eq. (8) is given. DGLAP kernels were taken from Ref. [14]. The full DGLAP kernel  $\Delta P$  includes both the LO and NLO terms:

$$\Delta P = \Delta P_{LO} + \widetilde{\alpha}_s(Q^2) \Delta P_{NLO}. \quad (15)$$

In the homogeneous term,  $\widetilde{\alpha}_s(Q^2) \int_{k_0^2}^{Q^2} \frac{dk^2}{k^2} (\Delta P_{reg} \otimes f_{NS})(x, k^2)$ , appearing in (8), only the regular part of the full kernel is included. This is to avoid double counting the NLO terms and  $\ln^2(1/x)$  terms in the region of the phase-space,  $k_0^2 < k^2 < Q^2$ .

Ladder kernels corresponding to the LO DGLAP kernels taken at  $z = 0$  [4] generate double logarithmic corrections in the region of  $Q^2 < k^2 < Q^2/z$ .

Non-ladder kernels were obtained in Ref. [4] from the infrared evolution equations for the singlet partial waves  $\mathbf{F}_0, \mathbf{F}_8$  [2, 3, 12, 13]. In [4] we noticed that extending the kernel of the double logarithmic evolution equations from the ladder one,

$$\widetilde{\alpha}_s(Q^2) \Delta P_{qq}/\omega, \quad (16)$$

to the modified one,

$$\widetilde{\alpha}_s(Q^2) \left( \Delta P_{qq}/\omega - (\mathbf{F}_8(\omega) \mathbf{G}_0)_{qq}/(2\pi^2\omega^2) \right), \quad (17)$$

gave a proper anomalous dimension as derived from the infrared evolution equations.

Matrix  $\mathbf{G}_0$  contained colour factors resulting from attaching the soft gluon to external legs of the scattering amplitude :

$$\mathbf{G}_0 = \begin{pmatrix} \frac{N^2-1}{2N} & 0 \\ 0 & N \end{pmatrix}, \quad (18)$$

where  $N$  was the number of colours.

Further, it was checked that the Born approximation of  $\mathbf{F}_8$ ,

$$\mathbf{F}_8^{Born}(\omega) \approx 8\pi^2 \widetilde{\alpha}_s(Q^2) \frac{\mathbf{M}_8}{\omega}. \quad (19)$$

gave accurate results for the DL evolution. Matrix  $\mathbf{M}_8$  was a splitting function matrix in colour octet  $t$ -channel,

$$\mathbf{M}_8 = \begin{pmatrix} -\frac{1}{2N} & -\frac{N_F}{2} \\ N & 2N \end{pmatrix}. \quad (20)$$

The inverse Mellin transform of  $\mathbf{F}_8^{Born}(\omega)$  then read :

$$\left[ \frac{\tilde{\mathbf{F}}_8^{Born}}{\omega^2} \right](z) = 4\pi^2 \widetilde{\alpha}_s(Q^2) \mathbf{M}_8 \ln^2(z). \quad (21)$$

The evolution equation (8) includes the non-ladder corrections in the Born approximation (21).

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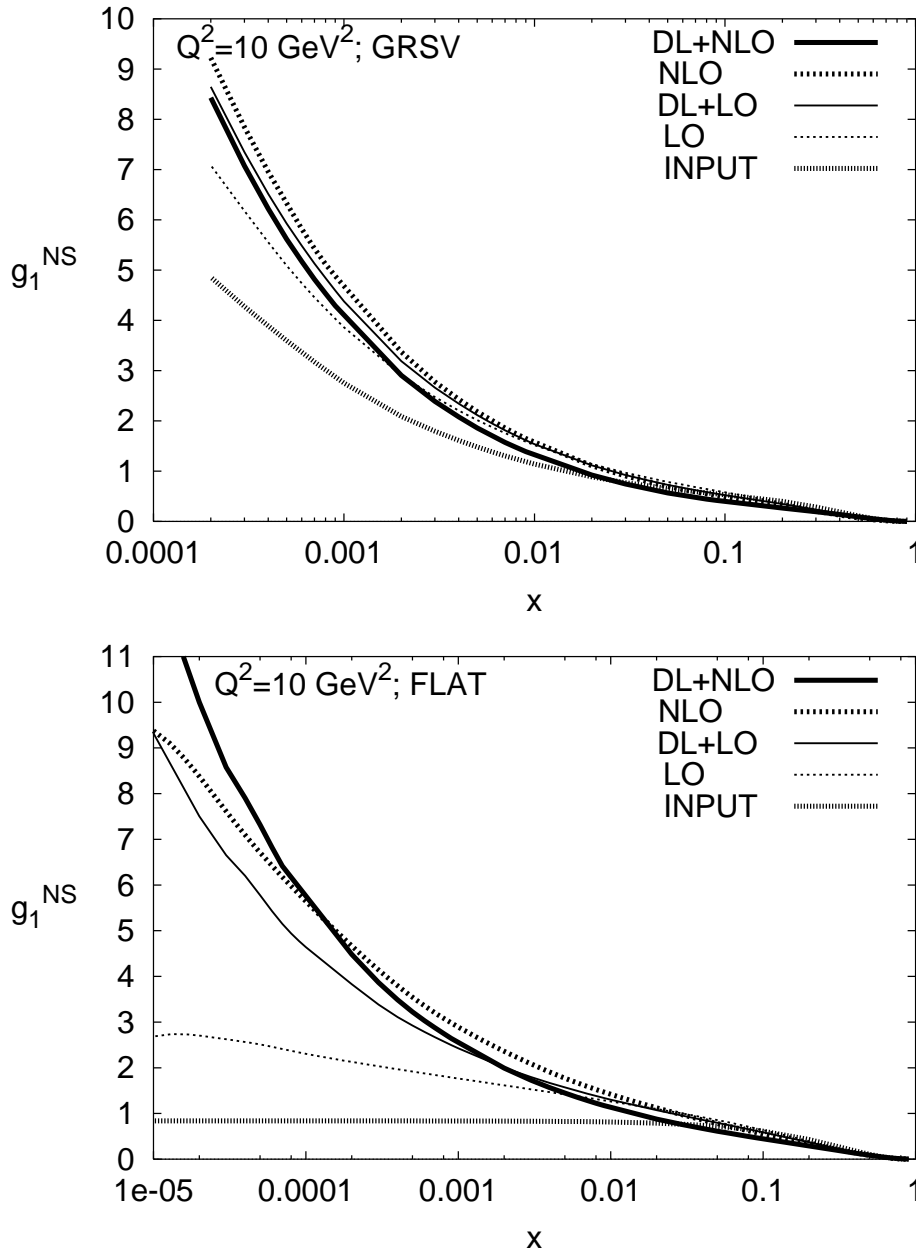


Figure 1: Non-singlet component of the nucleon structure function,  $g_1^{NS} = g_1^p - g_1^n$ , plotted as a function of  $x$  at the fixed  $Q^2 = 10 \text{ GeV}^2$ . Thick solid line (**DL+NLO**) shows the results obtained by: (i) numerical solving Eq. (8) for  $f_{NS,3}$  with the input given (thick dotted line), (ii) numerical integration of  $f_{NS,3}$  performed in order to obtain  $\Delta q_{NS,3}$ , and (iii) numerical convolution of  $\Delta q_{NS,3}$  with the regular part of the Wilson coefficient function (Eq. (14)). Thick dashed line (**NLO**) shows the NLO DGLAP evolution of the input. Thin lines correspond to the results: from the  $\ln^2(1/x)$  resummation including DGLAP terms at the LO accuracy (solid line, **DL+LO**), and from the pure LO DGLAP evolution (dashed line; **LO**). Upper plot shows the results from the GRSV'2000 input [18] ( $x > 10^{-4}$ ), lower plot shows the results from the flat input (12) ( $x > 10^{-5}$ ).

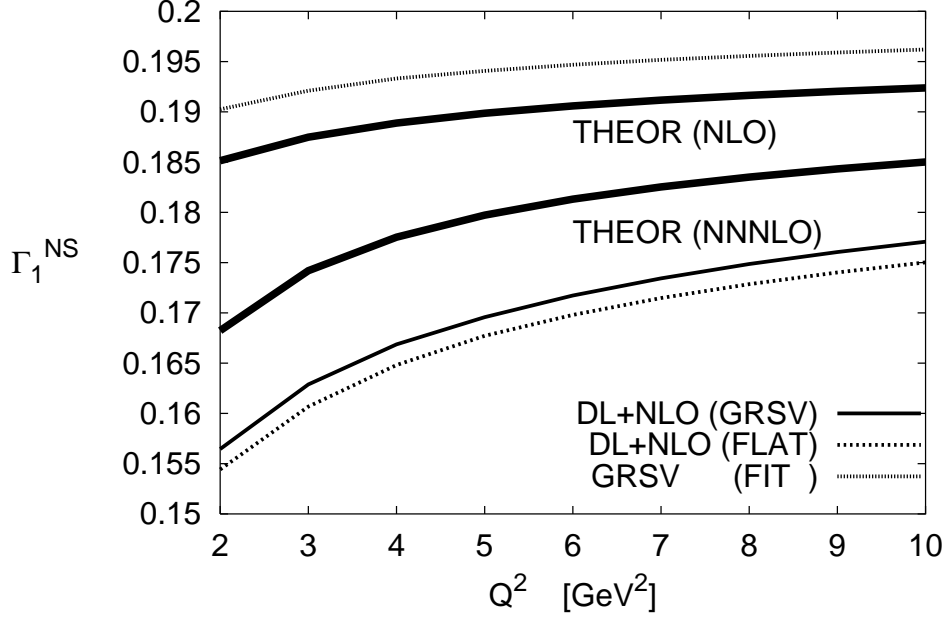


Figure 2: First moments of the non-singlet component of the nucleon structure function,  $g_1^{NS} = g_1^p - g_1^n$ , as a function of  $Q^2$  ( $2 < Q^2 < 10$  GeV<sup>2</sup>). Thin solid line shows the results from the GRSV non-perturbative input, dashed line shows the results from the flat input (12). They are compared with the theoretical predictions for the Bjorken sum rule (thick solid lines) calculated at the NLO and NNNLO accuracy [14], and with the first moment estimated from integrating out the NLO GRSV'2000 fit [18] (dotted line).